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A Remark on the Stabilization of Homogeneous Polynomial Systems

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Abstract—The purpose of this note is to re-establish, with a new and simple proof, a theorem which in fact represents the main results of [1].

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The goal of this note is to propose a new proof for results established in [1]. These results concern the stabilization of homogeneous polynomial systems.

Consider

$$\dot{x} = X(x) + \sum_{i=2}^{n} u_i Y_i(x), \tag{1}$$

with $X(x) = a(x)\frac{\partial}{\partial x_1}$ $(i = 1, \ldots, m)$, where $a : \mathbb{R}^n \longrightarrow \mathbb{R}$ is an homogeneous polynomial.

The results of the two main theorems of [1] can be stated as follows.

THEOREM 1. *A necessary and sufficient condition for the stabilization of (1) is there exist $\lambda, \mu \in \mathbb{R}^{n-1}$ such that $a(1, \lambda) < 0$ and $a(-1, \mu) > 0$.*

PROOF. Clearly that the condition of Theorem 1 is a necessary condition. It remains only to prove the sufficient part. Assume, without any loss of generality, using a suitable change of coordinates and a preliminary feedback if necessary, that

$$\lambda = (\lambda_1, 0, \ldots, 0) \quad \text{and} \quad \mu = (\mu_1, 0, \ldots, 0). \tag{2}$$

Indeed, if λ and μ, are not in the above form, we can use the following change of coordinates:
If $\lambda = -\mu$,

$$\begin{aligned} X_1 &= x_1, \\ X_i &= x_i - \lambda_{i-1} x_1, \qquad \text{for } i \in \{2, \ldots, n\}, \end{aligned} \tag{3}$$

and if $\lambda \neq -\mu$

$$\begin{aligned} X_1 &= x_1, \\ X_{i_0} &= x_{i_0}, \\ X_i &= x_i - \lambda_{i-1} x_1 - \frac{x_{i_0} - \lambda_{i_0-1} x_1}{\lambda_{i_0-1} + \mu_{i_0-1}} \left(\lambda_{i-1} + \mu_{i-1}\right), \qquad \text{for } i \in \{2, \ldots, n\} - \{i_0\}, \end{aligned} \tag{4}$$

where i_0 is such that $\lambda_{i_0-1} \neq -\mu_{i_0-1}$.

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Since (2), to prove Theorem 1, it suffices to give the proof for $n = 2$, and using the result of [2] we can conclude for the case $n > 2$. Hence, from now on, we assume that $n = 2$.

A simple reasoning shows, using a suitable change of coordinates and a preliminary feedback if necessary, that

$$a(x) = \alpha x_1^{s_0} (a_1 x_1 + x_2)^{s_1} \cdots (\alpha_n x_1 + x_2)^{s_n} \bar{a}(x),$$

with $\bar{a}(x) > 0$ for $x \neq 0$ and $\alpha_i \in \mathbb{R}^*$ such that $\alpha_i \neq \alpha_j$ if $i \neq j$.

We discuss two cases according to the value of s_i ($i = 1, \ldots, n$). First, assume that all s_i ($i = 1, \ldots, n$) are even. Then, since the condition of Theorem 1, we have $\alpha < 0$, and s_0 is odd.

Let

$$u_1(x) = -x_2, \tag{5}$$

and

$$V_1(x) = \frac{1}{2} \left(x_1^2 + x_2^2\right).$$

Straightforward computation shows that $\dot{V}_1(x) < 0$, $\forall x \neq 0$. Therefore, the closed-loop system (1),(5), is globally asymptotically stable at the origin.

Now, assume, without any loss of generality, that s_1 is odd.

Let

$$V_2(x) = \frac{1}{2} x_1^2 + \frac{1}{s_1 + 1} (\alpha_1 x_1 + x_2)^{s_1+1},$$

and

$$u_2(x) = -\alpha x_1^{s_0} (\alpha_2 s_1 + x_2)^{s_2} \cdots (\alpha_n x_1 + x_2)^{s_n} \bar{a}(x) \frac{\partial V_2}{\partial x_1}(x) - \frac{\partial V_2}{\partial x_2}(x), \tag{6}$$

we have

$$\dot{V}_2(x) = -\left(\frac{\partial V_2}{\partial x_2}(x)\right)^2 \leq 0 \qquad \forall x \in \mathbb{R}^2.$$

Consequently, the closed-loop system (1),(6) is stable. To establish the global asymptotic stability at the origin, we use the LaSalle Principle Invariance.

Let $x^0 \in \mathbb{R}^2$ such that $\dot{V}_2(x^0) = \frac{d\dot{V}_2(x^0)}{dt} = 0$. Then, x^0 verifies the following system

$$\alpha_1 x_1^0 + x_2^0 = 0,$$
$$x_1^0 \prod_{2 \leq i \leq n} (\alpha_i x_1^0 + x_2^0) = 0,$$

which, clearly implies $x^0 = 0$. This finishes the proof of Theorem 1.

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